university of groningen

Changing Individuals

Modelling smooth and sudden changes in temporal dynamics

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The basic AR(1) model

- Series of (psychological) measurements $y_1, \ldots, y_T$.
- Simplest form of the model:

$$y_t = \mu + \phi y_{t-1} + \varepsilon_t, \quad (t = 2, \ldots, T)$$

- Assume (for now) stationarity ($|\phi| < 1$) and $\varepsilon_i \sim N(0, \sigma^2)$

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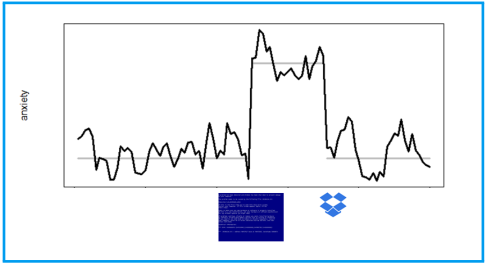
- Assume (for now) stationarity ($|\phi| < 1$) and $\varepsilon_i \sim N(0, \sigma^2)$
- Here, μ and ϕ are fixed: they can't change.
- But people do change.

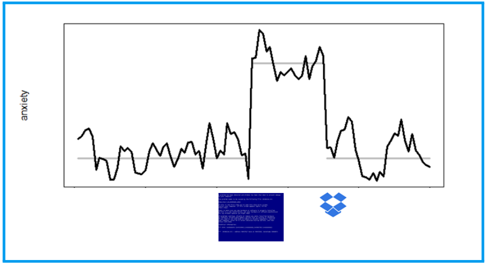
Two types of change in AR(1) models

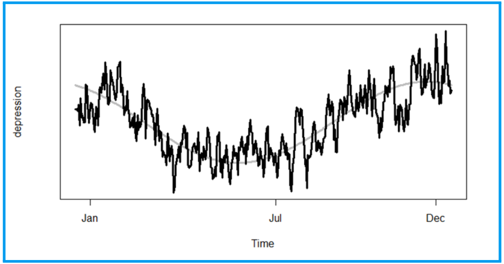
1. Sudden change



Two types of change in AR(1) models

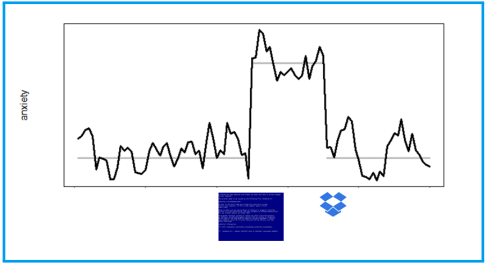
1. Sudden change
2. Smooth change





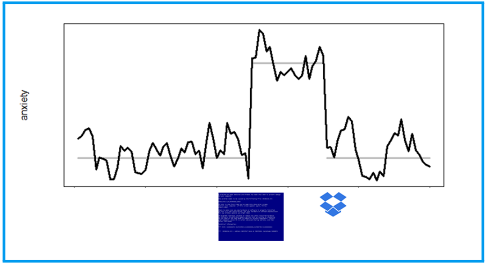
Two types of sudden change

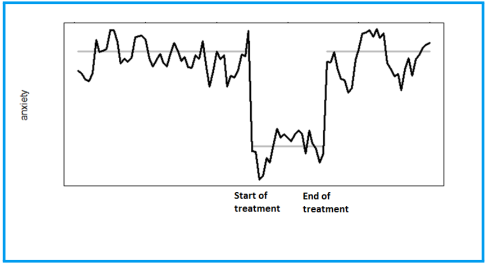
1. Sudden change at a unknown moment



Two types of sudden change

1. Sudden change at a unknown moment
2. Sudden change at an known moment





Goal of this talk

Thus, the dynamics in an AR(1) model can change

- Suddenly – at known moment(s)
- Suddenly – at unknown moment(s)
- Smoothly – (all the time)

(Many) models for one of these cases already exist.

My goal of the day: to introduce a model that combines all three cases.

Models for sudden change

Many different models exist, e.g.

- Markov switching (regime change) models (next slide)
- Models from in Statistical Quality Control (e.g. the CUSUM procedure; Page, 1954)
- Models from Deep Learning (e.g. Krylov subspace models; Ide & Tsuda, 2007)
- Models from Machine Learning (e.g. relative density-ratio method; Sugiyama, Suzuki, & Kanamori, 2012)
- . . . . (really, a *lot* of alternatives)

Regime Switching Models

- Use dummy-variable

$$D_{i,t} = \begin{cases} 0 & \text{in regime A at time } t < i \\ 1 & \text{in regime B at time } t \geq i \end{cases}$$

for some i.

Regime Switching Models

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- Then apply model

$$\begin{cases} y_t = \mu_{D_{0,t}} + \phi_{D_{0,t}} y_{t-1} + \varepsilon_t & \text{Regime A} \\ y_t = \mu_{D_{1,t}} + \phi_{D_{1,t}} y_{t-1} + \varepsilon_t & \text{Regime B} \end{cases}$$

  (cf. Hamilton, 1989)

- Straightforward if i known. Apply HMM to find i when unknown.

Model for smooth change

For this, we use the Time-Varying Autoregressive Model (TV-AR) by Bringmann et al. (Psychological Methods, 2017).

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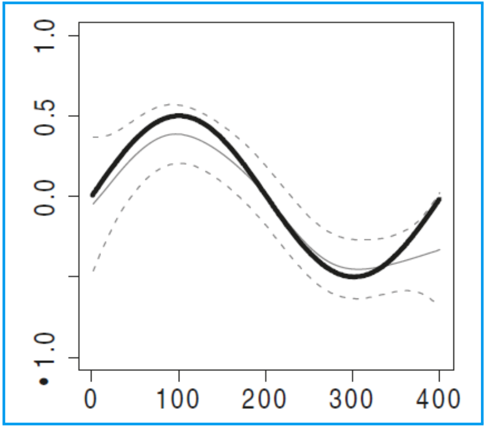
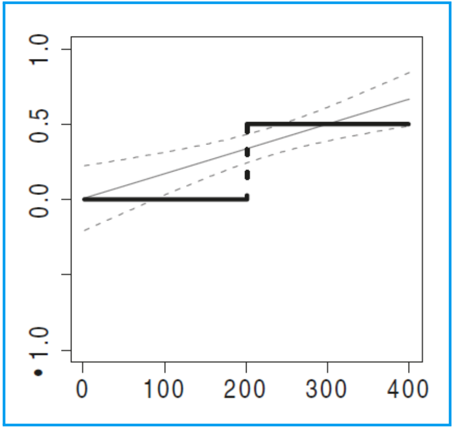
$$y_t = \mu_t + \phi_t y_{t-1} + \varepsilon_t$$

- μ_t and ϕ_t not fixed, yet are only allowed to vary smoothly: $\mu_t \approx \mu_{t+1}$ and $\phi_t \approx \phi_{t+1}$
- This is achieved by using Generalized Additive Models (Hastie & Tibshirani, 1990).

TV-AR model for smooth change





works great doesn’t work

Our model – confirmatory analyses

Basic idea of our TV-AR-RS model:

Combine TV-AR's smooth parameters with Hamilton's RS idea:

$$y_t = (\mu_t + \mu_{D_{i,t}}) + (\phi_t + \phi_{D_{i,t}}) \times y_{t-1} + \varepsilon_t$$

(with $D_{i,t}$ a 0/1-variable)

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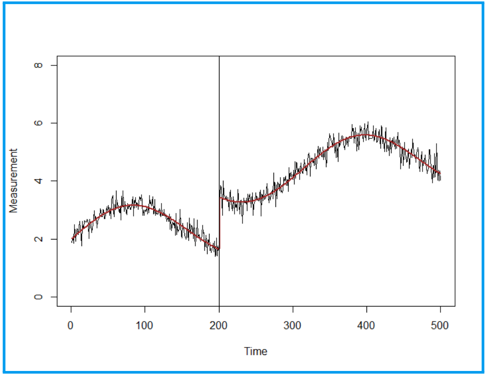
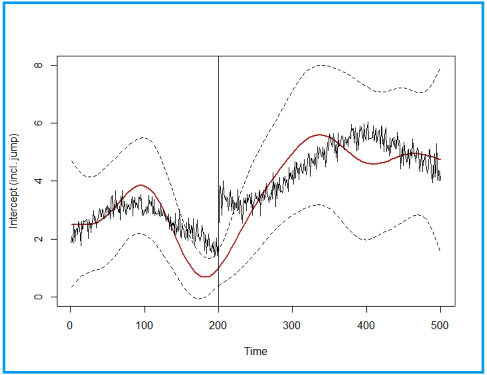
mcgv-package provides curves for μ_t and ϕ_t including CI, and point estimates for μ_D, ϕ_D including SE, and model fit statistics. All you need.

TV-AR-RS model – confirmatory analyses – example



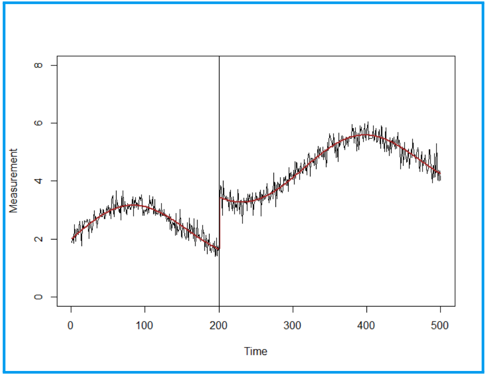
Simulated data



TV-AR model

TV-AR model: AIC = 173.42

TV-AR-RS model – confirmatory analyses – example



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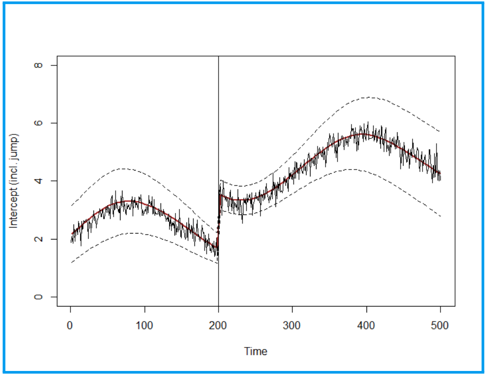
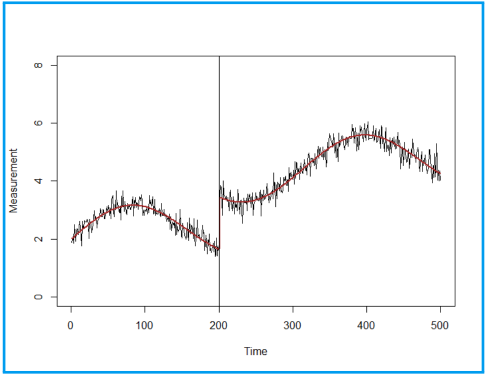


TV-AR-RS with correct jump

TV-AR model: AIC = 173.42

Correct TV-AR-RS model: AIC = 64.63, $\hat{\mu}_1 = 1.88$ (sd=.13)

TV-AR-RS model – confirmatory analyses – example



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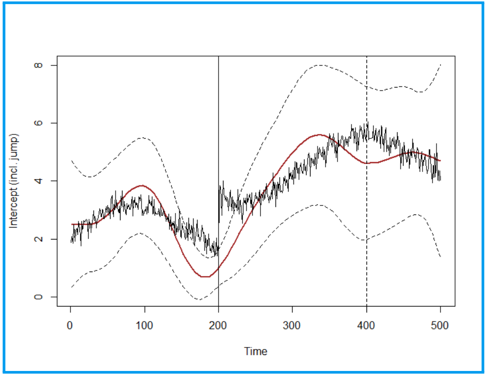

TV-AR-RS with correct jump

TV-AR model: AIC = 173.42

Correct TV-AR-RS model: AIC = 64.63, $\hat{\mu}_1 = 1.88$ (sd=.13)

Incorrect TV-AR-RS model: AIC = 175.27, $\hat{\mu}_1 = .04$ (sd=.13)

TV-AR-RS model – exploratory analyses

Sketch of the algorithm:

1. Compute $\text{AIC}^{(0)}$ for model $y_t = \mu_t + \phi_t \times y_{t-1} + \varepsilon_t$

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4. IF $AIC_i^{(1)} < AIC^{(0)} - 10$ THEN select point i as new change point ELSE stop

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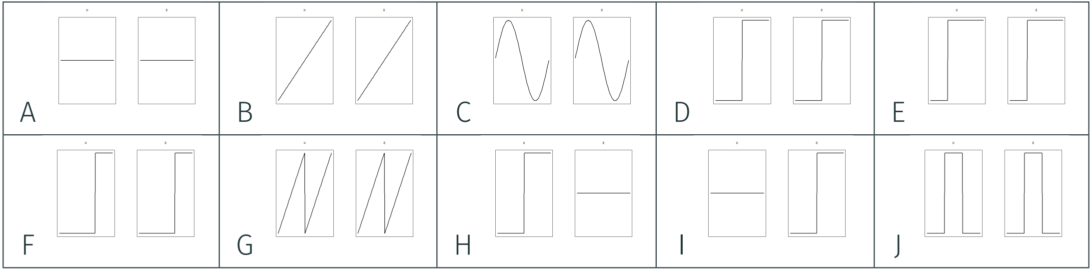
3. Select $i = \text{argmin}_j\, \text{AIC}_j^{(1)}$
4. IF $AIC_i^{(1)} < AIC^{(0)} - 10$ THEN select point i as new change point ELSE stop
5. Re-run steps 2 – 4 to find subsequent change points.

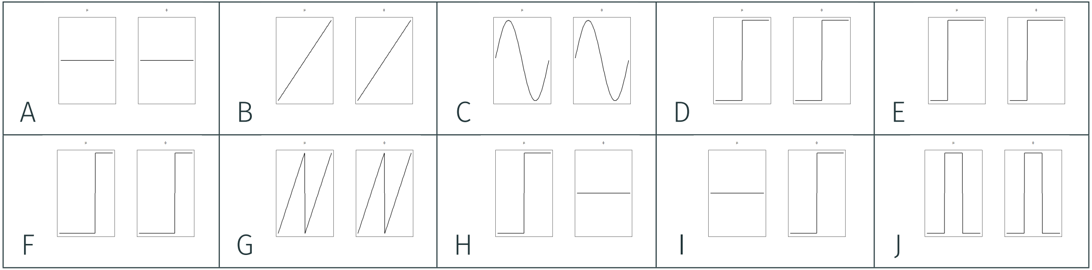
(If desired, replace AIC by BIC or any other fit measure.)

Multiple conditions:



Simulation: design

Multiple conditions:



Furthermore:

- Small or large regime switches
- Length of time series: $T = 30, 60, 100, 200, 400, 600, 1000$

Fully crossed design, 100(0) replications per cell.

Simulation: performance measures

I. How much better (or worse) is the model with correct change point, compared to model without change point?

$$\overline{AIC_j^{(1)} - AIC^{(0)}}$$

Simulation: performance measures

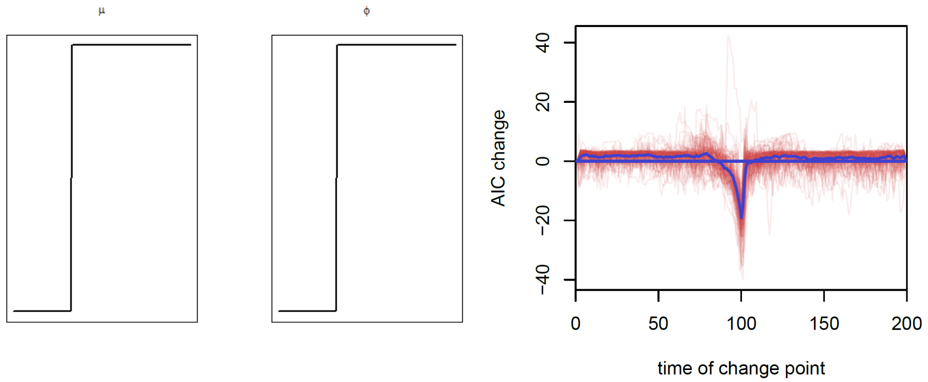
I. How much better (or worse) is the model with correct change point, compared to model without change point?

$$\overline{AIC_j^{(1)} - AIC^{(0)}}$$

II. Is the change point placed at the right location?

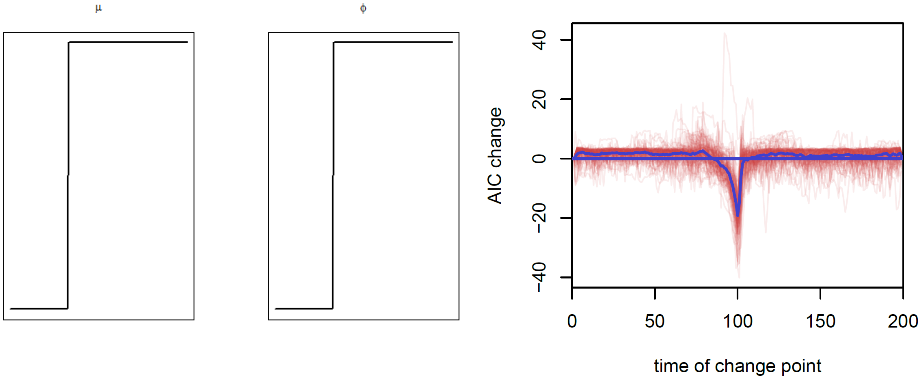
$$Median|i - j|$$

Simulation: example



$T = 200$, change in μ: 2sd, change in ϕ: 0.7. $R = 100$ replications.

Simulation: example



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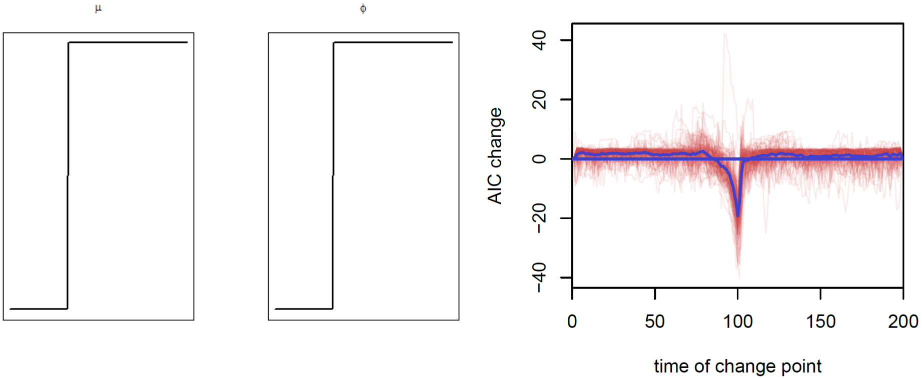
AIC gain at $J = 100$: $m = 18.89$, $s = 7.26$.

Simulation: example



$T = 200$, change in μ: 2sd, change in ϕ: 0.7. $R = 100$ replications.

AIC gain at $J = 100$: $m = 18.89$, $s = 7.26$.

12% of cases: jump at $T = 100$, 61%: jump at 99 or 101.

Simulation results

Mean AIC gain when the changes at the change point are small.

Cond.	$n = 30$	60	100	200	400	600	1000
A	1	2	2	2	2	2	2
B	1	2	2	2	2	2	2
C	1	1	1	1	2	2	2
D	1	-1	-2	-5	-10	-14	-23
E	-0	-0	-2	-5	-11	-16	-22
F	-0	-1	-3	-5	-10	-15	-23
G	-1	-1	-3	-7	-12	-14	-20
H	-0	-1	-2	-3	-6	-10	-14
I	1	2	1	1	-1	-2	-4
J(L)	0	-1	-2	-4	-7	-9	-16
J(R)	-1	-1	-1	-2	-5	-9	-13

Simulation results

Mean AIC gain when the changes at the change point are large.

Cond.	$n = 30$	60	100	200	400	600	1000
A	0	1	1	1	2	2	2
B	1	2	2	2	2	2	2
C	0	1	1	2	2	2	2
D	-1	-5	-10	-21	-33	-45	-66
E	-1	-4	-9	-19	-34	-44	-66
F	-0	-5	-11	-21	-32	-42	-62
G	-10	-21	-27	-40	-57	-67	-85
H	-1	-3	-5	-11	-16	-22	-29
I	1	-2	-6	-16	-25	-34	-55
J(L)	-1	-3	-6	-11	-23	-32	-51
J(R)	-17	-25	-4	-12	-13	-57	-41

Simulation results

Median absolute 'miss' ($|i - j|$) when the changes at the change point are small.

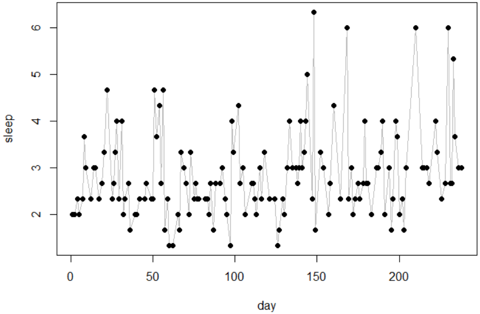
Cond.	$n = 30$	60	100	200	400	600	1000
D	5	10.5	22.5	2.5	2	2	1
E	7	11	18	7	2	2	1.5
F	6	9	8	4	2	2	1
G	5	10.5	6.5	1.5	1.5	2	2
H	5.5	9	20	22.5	4	2	3
I	7	14.5	26	49.5	106.5	67	69.5

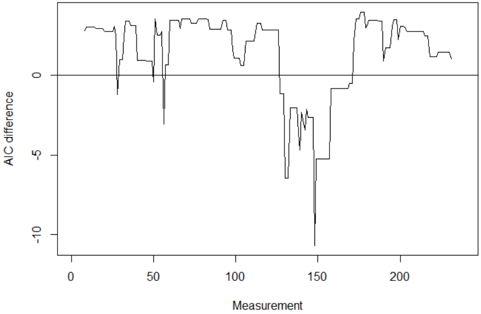
Simulation results

Median absolute 'miss' ($|i - j|$) when the changes at the change point are large.

Cond.	$n = 30$	60	100	200	400	600	1000
D	6	5	1	1	1	1	1
E	5	7.5	2	1	1	1	1
F	8.5	4	1	1	1	1	1
G	2	1	1	1	1	1	1
H	6	9.5	13	1	1	1	1
I	7	10.5	2.5	1	1	1	1

Example: $n = 1$ sleep quality data





At day 148 significant jump:
AIC-gain 10.67 points. Change in μ ($p = .038$) *and* change in ϕ ($p = .026$).

Conclusions

- Our VAR model can deal with smooth and sudden change in dynamics.
- Changes in ϕ harder to detect than those in μ.
- Can be used for both confirmatory and exploratory purposes.
- Model works, but only for sufficiently large data sets:
    - Large 'jump': at least $T > 60$
    - Small 'jump': at least $T > 200$

Key references:

- Albers & Bringmann (2018). Changing Individuals. In preparation.
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- Hamilton (1989). A new approach to the economic analysis of nonstationary time series and the business cycle, Econometrica
- Kossakowski, Groot, Haslbeck, Borsboom, Wichers (2017). Journal of Open Psychology Data